

## Appendix W7.9

# Digital Implementation of Example 7.33

### EXAMPLE W7.2

#### *Servomechanism*

For Example 7.33, derive an equivalent discrete controller with a sampling period of  $T_s = 0.1$  sec ( $20 \times \omega_n = 20 \times 0.05 = 0.1$  sec), and compare the continuous and digital control outputs, as well as the control efforts. Verify the design by plotting the step response and commenting on the comparison of the continuous and discrete responses.

**Solution.** The discrete equivalent for the controller is obtained from Matlab by using the c2d command, as in

Matlab c2d

```
nc=conv([1 1],[8.32 0.8]); % controller numerator
dc=conv([1 4.08],[1 0.0196]); % controller denominator
sysDc=tf(nc,dc); % form controller system description
ts=0.1; % sampling time of 0.1 sec
sysDd=c2d(sysDc,ts,'zoh'); % convert to discrete time controller
```

The discrete controller has the discrete transfer function

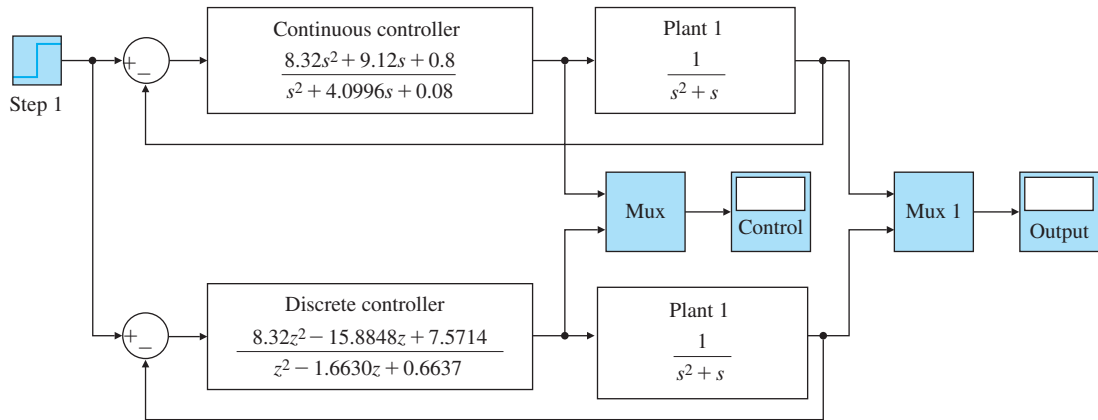
$$D_d(z) = \frac{8.32z^2 - 15.8855z + 7.5721}{z^2 - 1.6630z + 0.6637} = \frac{8.32(z - 0.9903)(z - 0.9191)}{(z - 0.998)(z - 0.6665)}.$$

The equation for the control law (with sample period suppressed for clarity) is

$$u(k+1) = 1.6630u(k) + 0.6637u(k-1) + 8.32e(k+1) - 15.8855e(k) + 7.5721e(k-1).$$

Simulink simulation

A Simulink diagram for simulating both the continuous and discrete systems is shown in Fig. W7.3. A comparison of the continuous and discrete step responses and control signals is shown in Fig. W7.4. Better agreement between the two responses was achieved because the sample rate was 20 times the bandwidth for this example.



**Figure W7.3**

Simulink block diagram to compare continuous and discrete controllers

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**Figure W7.4**

Comparison of step responses and control signals for continuous and discrete controllers: (a) step responses; (b) control signals

