

Appendix W5.4.4

Analog and Digital Implementations

Lead compensation can be physically realized in many ways. In analog electronics, a common method is to use an operational amplifier, an example of which is shown in Fig. W5.1. The transfer function of the circuit in Fig. W5.1 is readily found by the methods of Chapter 2 to be

$$D_{lead}(s) = -a \frac{s+z}{s+p}, \quad (\text{W5.1})$$

where

$$a = \frac{p}{z}, \quad \text{if } R_f = R_1 + R_2,$$

$$z = \frac{1}{R_1 C},$$

$$p = \frac{R_1 + R_2}{R_2} \cdot \frac{1}{R_1 C}.$$

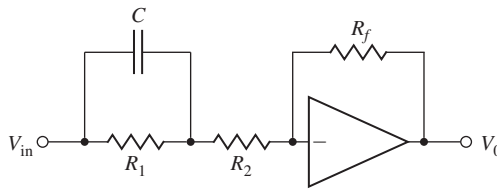
If a design for $D_c(s)$ is complete and a digital implementation is desired, then the technique of Appendix W4.5 can be used by first selecting a sampling period T_s and then making substitution of $\frac{2}{T_s} \frac{z-1}{z+1}$ for s . For example, consider the lead compensation $D_c(s) = \frac{s+2}{s+13}$. Then, since the rise time is about 0.3, a sampling period of six samples per rise time results in the selection of $T_s = 0.05$ sec. With the substitution of $\frac{2}{0.05} \frac{z-1}{z+1}$ for s into this transfer function, the discrete transfer function is

$$\begin{aligned} \frac{U(z)}{E(z)} &= \frac{40 \frac{z-1}{z+1} + 2}{40 \frac{z-1}{z+1} + 13} \\ &= \frac{1.55z - 1.4}{1.96z - 1}. \end{aligned} \quad (\text{W5.2})$$

Clearing fractions and using the fact that operating on the time functions $zu(kT_s) = u(kT_s + T_s)$, we see that Eq. (W5.2) is equivalent to the formula for the controller given by

Figure W5.1

Possible circuit of a lead compensation



$$u(kT_s + T_s) = \frac{1}{1.96}u(kT_s) + \frac{1.55}{1.96}e(kT_s + T_s) - \frac{1.4}{1.96}e(kT_s). \quad (W5.3)$$

The Matlab commands to generate the discrete equivalent controller are

```
sysC=tf([1 2],[1 13]);
sysD=c2D(sysC,0.05,'tustin')
```

Fig. W5.2 shows the Simulink diagram for implementing the digital controller. The result of the simulation is contained in Fig. W5.3, which shows the comparison of analog and digital control outputs, and Fig. W5.4, which shows the analog and digital control outputs.

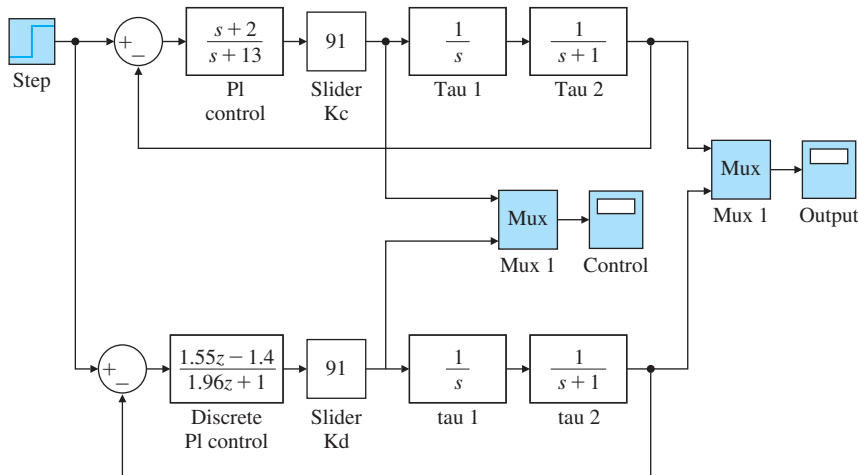


Figure W5.2

Simulink diagram for comparison of analog and digital control

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Figure W5.3

Comparison of analog and digital control output responses

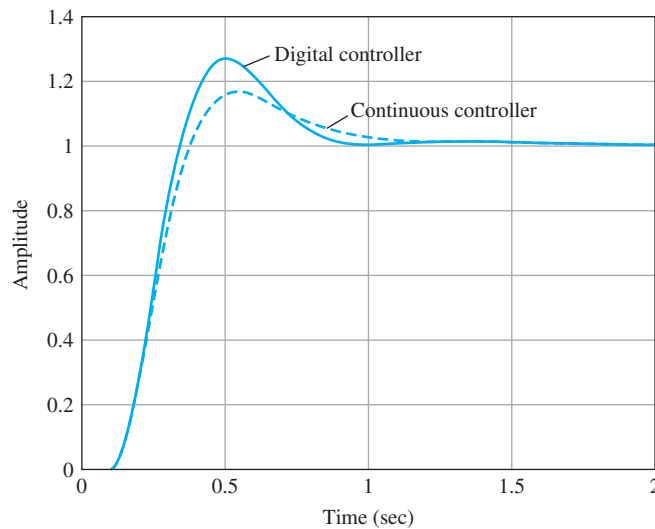
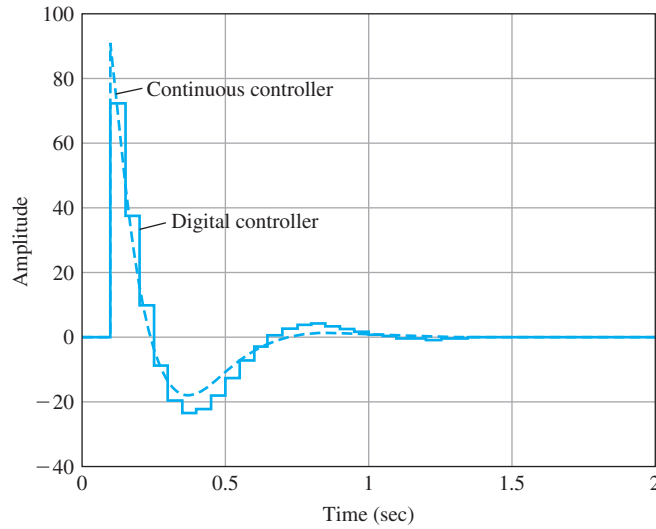
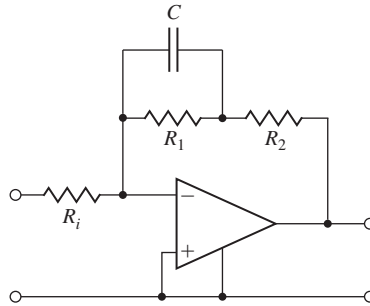


Figure W5.4

Comparison of analog and digital control time histories

**Figure W5.5**

Possible circuit of lag compensation



As with lead compensation, lag or notch compensation can be implemented using a digital computer and following the same procedure. However, they too can be implemented using analog electronics, and a circuit diagram of a lag network is given in Fig. W5.5. The transfer function of this circuit can be shown to be

$$D_c(s) = -a \frac{s+z}{s+p},$$

where

$$a = \frac{R_2}{R_i},$$

$$z = \frac{R_1 + R_2}{R_1 R_2 C},$$

$$p = \frac{1}{R_1 C}.$$

Usually $R_i = R_2$, so the high-frequency gain is unity, or $a = 1$, and the low-frequency increase in gain to enhance K_v or other error constant is set by $k = a \frac{z}{p} = \frac{R_1 + R_2}{R_2}$.