INTEGRATED CONTROL: A DECENTRALIZED APPROACH

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ABSTRACT

A decentralized, robust, multivariable controls method is described for the functional integration of subsystems in large-scale systems characterized by dynamic coupling among subsystem elements. In an integrated environment, a decentralized control structure reduces the complexity of the control by distributing the control authority to local controllers. The method relies upon generation of a set of control design specifications for the subsystems. The subsystem designers are provided with design specifications that provide for the integration of control functions. If the subsystem designers can satisfy these specifications, successful integration of the subsystem controls is guaranteed. The specifications are in terms of stability and performance $% \left(1\right) =\left\{ 1\right\} =\left\{ 1$ robustness constraints in the frequency domain. The results are applied to an integrated flight-propulsion control example.

I. INTRODUCTION

A decentralized robust multivariable controls approach is developed for integrated control. The basic philosophy is that in an integrated controls environment, a decentralized control structure reduces the complexity of the control by distributing the control authority to local controllers. This allows the system to be decomposed into simpler comprehensible subsystems with simpler design problems. The logical decomposition into subsystems makes the design comprehensible at the global level and optimal at the local (subsystem) level. Comprehensibility dictates a hierarchical structure, with simple decentralized goals at each level of the hierarchy. A decentralized design also suggests a decentralized architecture which is quite desirable. Integrated control will coordinate various system functions associated with different subsystems and holds the promise of reducing human operator workload while improving the performance and reliability/maintainability of the system.

Consider the integrated linear dynamic system

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du (2)$$

which is the interconnection of $\ensuremath{\mathtt{m}}$ coupled linear dynamic subsystems:

$$\dot{x}_{i} = A_{ii} x_{i} + B_{ii} u_{i} + \sum_{j \neq i} (A_{ij} x_{j} + B_{ij} u_{j}) \qquad i = 1, ..., m$$

$$y_{i} = C_{ii} x_{i} + D_{ii} u_{i} + \sum_{j \neq i} (C_{ij} x_{j} + B_{ij} u_{j}) \qquad (3)$$

$$+ D_{ij} u_{j}$$
) $i = 1, ..., m$ (4)

where the subscript i refers to the ith subsystem. These equations may be rewritten in the form:

$$\dot{x}_{i} = \overline{A}_{ij} x_{i} + \overline{B}_{ij} u_{i} + \sum_{j \neq i} \Gamma_{ij} y_{j}$$
 (5)

$$y_{i} = \overline{C}_{ii} x_{i} + \overline{D}_{ii} u_{i} + \sum_{j \neq i} W_{ij} y_{j}$$
 (6

i.e., the equations are driven by outputs of other subsystems. Note that selecting a set of y_j 's that yield physically meaningful equations (5) and (6) can be a nontrivial task. Equations (1) and (2) or (5) and (6) imply the decomposition of the system into m subsystems and correspond to partitioning the transfer function matrix as

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \vdots & \dots \\ G_{21}(s) & G_{22}(s) & & \dots \\ \vdots & \vdots & \ddots & \end{bmatrix}$$
(7)

where

$$G(s) = C(sI - A)^{-1}B + D$$
 (8)

The decentralized compensator is of the form

$$K(s) = diag \{K_{11}(s), K_{22}(s), ..., K_{mm}(s)\}$$
 (9)

where $K_{ij}(s)$ is of size $k_i \times k_j$ and

$$\begin{array}{ccc}
\mathbf{m} \\
\mathbf{\Sigma} & \mathbf{k}_{1} = \mathbf{n} \\
\mathbf{i} = 1
\end{array} \tag{10}$$

The problem is to design the controllers $K_{ij}(s)$ so that the closed-loop system is stable and the global performance objectives are achieved.

Many steps are involved in carrying out this decentralized control design. Assuming that overall linear models have been obtained, the system needs to be partitioned into subsystems (as described by (5) and (6)) using appropriate techniques. As far as the actual control design is concerned, the performance goals should be translated into control requirements. The control requirements are then

mapped into specifications on tracking, disturbance rejection, etc. If an LQG methodology is followed, the performance index for the regulator should include weights on state deviations, tracking error, and other subsystem variables which affect the global response of the system. It need not include factors specific to subsystem designs and unimportant to overall system performance. This is referred to as the high-level design. The models in the high-level design should be sufficiently detailed so as to include every effect that is weighted in the high-level design directly influencing the global response. The high-level regulator may then be translated into an equivalent output feedback form for ease of implementation.

The results of the high-level design specify a closed-loop description of how every subsystem must perform in accomplishing the control integration. Note, however, that the high-level design optimizes a global performance index. It does not take into account all of the specific operating constraints and limitations of the subsystems. Subsystem control designs that yield global closed-loop performance while satisfying many additional subsystem specific constraints are required to complete the integrated design. The main thrust of this paper is to discuss the purpose and form of subsystem design specifications which would guarantee successful integration of the subsystem designs.

A feature of the proposed integrated control design technique is that the subsystem controls (i.e., $K_{ij}(s)$) can be designed separately. Each subsystem design is a well-defined multivariable control problem. Design specifications are generated for each subsystem that provide for the integration of the control functions. Hence, if the subsystem control designers can satisfy the specifications, successful integration of the subsystem control will be guaranteed. The subsystem controls can then be done by subsystem manufacturers who have expert knowledge of the subsystems, and are therefore, in the best position to understand their limitations and capabilities.

The subsystem control designers must satisfy two types of specifications. One is the subsystem specific set with which they are intimately familiar. The other is a set that describes the closed-loop specifications dictated by the global control design.

All of this has to be done in spite of model uncertainty present in the system. The specifications must then be generated with respect to tolerances. Both the high-level and subsystem controls are to be designed using multivariable, robust control theory. This requires development of both stability and performance robustness measures, as well as subsystem interaction measures in the frequency domain.

In an integrated control system, there may be several ways to produce the same effect on the system using different combinations of actuators. Hence, the high-level design can be done in terms of "generalized" actuators:

$$\dot{x}_1 = A_1 x_1 + B_1^* u^*$$
 (11)

where u* are the generalized actuators. The highlevel regulator then generates inputs in terms of generalized control commands (see Section III). These generalized control commands would then have to be translated to commands to individual affectors:

$$u = T u^*$$
 (12)

The (dynamic) element which provides this interface, T, will be referred to as the <u>control selector</u>. The control selector serves many functions which include distribution of generalized control commands to various affectors, redistri- bution of control when an affector is saturated, and system reconfiguration in the event of actuator failure.

II. SUBSYSTEM SPECIFICATIONS

Once the high-level design has been done, there must be a framework by which the high-level control requirements (related to global objectives) are mapped into subsystem requirements, e.g., in terms of stability, tracking, and disturbance rejection. primary requirement of the subsystem specifications is that they must be in a form understandable to subsystem control designers. Candidate forms include bandwidth and accuracy requirements and disturbance rejection requirements. Two types of specifications have been considered. The first is essentially an accuracy/bandwidth requirement and the second is a compensator gain bound requirement. The first type of specification is familiar to control system designers and is in terms of constraints on the singular values of the loop transfer functions of the subsystems closed-loop models. second type of specification is the one that the control system designers are not so familiar with. These measures are a function of the open-loop dynamics, system partitioning and are related to interaction measures among subsystems defined in the frequency domain. They are constraints on the singular values of the compensator transfer func-

2.1 Performance Robustness Specification

Performance of a control system may be evaluated in terms of its transient response behavior, i.e., command following, disturbance rejection, etc. Performance robustness refers to the ability of the control system to maintain a given level of performance despite model uncertainty. Performance robustness measures developed here establish a quantitative interrelationship between performance and model uncertainty.

Consider the system with transfer function matrix

$$G(s) = G_{0}(I + \Delta)$$
 (13)

where G_0 is the nominal transfer function matrix and Δ represents the unstructured (input multiplicative) model uncertainty. In terms of state variable matrices

$$G_0(s) = C(sI-A)^{-1}B + D$$
 (14)

Assume that the performance requirements are expressed in the frequency domain in terms of

$$\frac{\|H_{YR}(\Delta) - H_{YR}\|}{\|H_{YR}\|} < \rho(\omega) , \quad \omega \ge 0$$
 (15)

where $\rho(\omega)$ represents some given function of frequency indicating specified level of performance degradation from nominal, and $H_{\mbox{\scriptsize YR}}$ is the nominal closed-loop input-output transfer function matrix.

The model uncertainty in the system, Δ , may be isolated using the method in [1] and the following closed-loop "interconnection" structure

$$\begin{bmatrix} Y \\ Z \end{bmatrix} = \begin{bmatrix} H_{YR} & H_{Yd} & H_{YV} \\ H_{ZR} & H_{Zd} & H_{ZV} \end{bmatrix} \begin{bmatrix} R \\ d \\ v \end{bmatrix}$$
 (16)

is associated with the feedback system. The variables Z and V represent the input and output to the isolated model uncertainty and d is an additive disturbance. Assume that both Δ and the transfer function matrices in (16) are stable linear time-invariant operators. Then the following performance robustness measure

$$\delta_{PR}(\omega) = \left[\|H_{ZV}\| \|H_{YR}\|_{\rho}(\omega) + \|H_{YV}\| \|H_{ZR}\| \right]^{-1}$$

$$\bullet \rho(\omega) \|H_{VR}\| \qquad (17)$$

is associated with the system in (13). The stability robustness measure is given by

$$\delta_{SR}(\omega) = \frac{1}{\|H_{7V}\|} \tag{18}$$

and

$$\delta_{pR}(\omega) < \delta_{SR}(\omega), \quad \omega \ge 0$$
 (19)

If $\delta(\omega)$ represents a bound on model uncertainty, i.e.

$$\|\Delta\| < \delta(\omega) , \quad \omega \ge 0 \tag{20}$$

then performance is guaranteed in spite of modeling errors, if

$$\delta(\omega) < \delta_{pR}(\omega) , \quad \omega \ge 0$$
 (21)

The above may be summarized as follows.

<u>Theorem 1</u>: Assume that the performance requirements are expressed in the form of (15) then the closed-loop system is stable if

$$\delta_{SR}(\omega) > \delta(\omega)$$
 (22)

Furthermore, if the system is stable, then performance requirements are guaranteed, provided

$$\delta_{\rm DR}(\omega) > \delta(\omega)$$
 (23)

with

$$\delta_{PR}(\omega) < \delta_{SR}(\omega)$$
 (24)

The performance measure in (17) can be specialized to the tracking performance. For the output feedback problem

$$u = K_{Y}(R-Y) \tag{25}$$

where \mbox{K}_{γ} is the output feedback gain matrix, \mbox{R} is the reference input, and \mbox{Y} is the sensed output we have that

$$H_{YR} = (I + G_0 K_Y)^{-1} G_0 K_Y$$
 (26)

$$H_{VV} = (I + G_0 K_V)^{-1} G_0$$
 (27)

$$H_{ZV} = -(I + K_Y G_0)^{-1} K_Y G_0$$
 (28)

$$H_{ZR} = (I + K_Y G_0)^{-1} K_Y$$
 (29)

and

$$\delta_{PR_{T}(\omega)} = \frac{\rho(\omega) \overline{\sigma}(H_{YR})}{\overline{\sigma}(H_{ZV})\overline{\sigma}(H_{YR})\rho(\omega) + \overline{\sigma}(H_{YV})\overline{\sigma}(H_{ZR})}$$
(30)

where $\sigma(\cdot)$ denotes the upper singular value. The $\delta_{SR}(\omega)$ and $\delta_{PR}(\omega)$ from the high-level design can then be used to obtain subsystem specifications.

Similarly, if the disturbance rejection performance robustness requirements are described by

$$||Y|| \le \beta(\omega) \quad , \quad \forall \omega \tag{31}$$

then, so long as

$$\delta < \delta_{PR_{D}}, \forall \omega$$
 (32)

where $\delta_{\mbox{\footnotesize{PR}}_{\mbox{\footnotesize{D}}}}$ is a disturbance rejection performance robustness measure

$$\delta_{PR_D}(\omega) = \frac{\beta - \overline{\sigma}(H_{Yd})}{\overline{\sigma}(H_{ZV})(\beta - \overline{\sigma}(H_{Yd})) + \overline{\sigma}(H_{YV})\overline{\sigma}(H_{Zd})}$$
(33)

then

$$||Y|| \le \beta \qquad \forall \omega \tag{34}$$

For the output feedback problem

$$H_{Yd} = (I + G_{o}K_{Y})^{-1}$$
 (35)

$$H_{ZV} = (I + K_Y G_0)^{-1} K_Y G_0$$
 (36)

$$H_{YV} = (I + G_0 K_Y)^{-1} G_0$$
 (37)

$$H_{7d} = (I + K_Y G_Q)^{-1} K_Y$$
 (38)

2.2 Stability Constraint Specification

The second type of specification relies upon a frequency domain treatment of the interaction between subsystems. A bound as a function of frequency can be found on the compensator transfer function matrix to provide a sufficient test for the stability of the decentralized system [2]. A condition is needed to assure the stability of the closed loop system when the subsystem designs have been carried out and integrated. Note that the nature of the m subsystems may be quite different and may require different multivariable control design approaches. Following Ref. 2, we define the ith (block) row interaction measure as

$$\gamma_{j}(\omega) \stackrel{\triangle}{=} \underline{\alpha} \{G_{jj}(j\omega)\} - \sum_{k=1}^{m} \sigma \{G_{jk}(j\omega)\}$$
 (39)

and the ith (block) column interaction measure as:

$$Y_{i}^{'}(\omega) = \underline{\sigma} \left\{ G_{ij}(j\omega) \right\} - \sum_{\substack{k=1\\k=i}}^{m} \sigma \left\{ G_{kj}(j\omega) \right\}$$
(40)

where $\underline{\sigma}\{\bullet\}$ and $\sigma\{\bullet\}$ denote the minimum and maximum singular values of the transfer function matrices involved.

<u>Theorem 2</u>: A sufficient condition for stability of the decentrally controlled closed-loop system is that:

- (1) there are no unstable fixed modes;
- (2) each subsystem is asymptotically stable by itself (i.e., with no coupling); and

(3)
$$\frac{1}{\sigma(K_{ij})} < \min_{\omega \in \Omega} \max \{ \gamma_i(\omega), \gamma_i'(\omega) \}$$
 (41)

where Ω denotes the bandwidth of interest.

Note that (41) provides a graphical test for constraints on

$$||K_{ij}(j\omega)|| \tag{42}$$

and the constraint may be conservative. Less conservative measures may be obtained using the results in [2-4]. Furthermore, the bound in (41) can be directly related to the bound on model error and is therefore quite consistent with robust control theory approaches.

III. EXAMPLE: INTEGRATED FLIGHT PROPULSION CONTROL FOR ADVANCED TECHNOLOGY, HIGH PERFORMANCE AIRCRAFT

Historically, advanced tactical aircraft design has been based on the philosophy that flight and propulsion controls could be designed separately. The pilot effectively integrated the independent systems by his control inputs. Recent advances in aircraft technology have greatly increased the coupling between the flight and propulsion systems, as well as the complexity of the aircraft controls. This coupling, combined with increased mission requirements, has made impractical the design of independent flight and propulsion controls which rely on the pilot for control integration. Instead, future advanced tactical aircraft will have an integrated control structure which will ensure the coordinated function of the subsystems by accounting for, and where possible, taking advantage of, the cross-coupling between system elements. In this example, we have applied the robust decentralized control design methodology to develop such a control structure for the integrated flight and propulsion control of an advanced tactical aircraft.

The advanced technology features which improve aircraft performance also increase the coupling between the two aircraft axes and between the flight and propulsion subsystems. Such enabling features included in the example aircraft are: variable geometry inlets, thrust vectoring/thrust reversing nozzles, cannards, and propulsive lift. These

features make the combined airframe/engine a coupled, high-order system.

For the purpose of demonstrating the decentralized control methodology, the aircraft was divided into five subsystems. The subsystems are the longitudinal aircraft, lateral aircraft, airflow, gas generator, and nozzle. These subsystems were chosen based on modal and controllability/observability properties. This choice of partitioning may not be optimal, but it demonstrates the effectiveness of the decentralized control design method because of cross-coupling between the subsystems.

The control structure for this example consists of an outer loop "flight management computer" and "maneuver command generator" which receive pilot commands and issue commands to the individual local subsystem controllers [5] (see Figure 1). For details of the control structure, see Ref. [6]. These local controllers must meet the subsystem performance specifications (e.g., tracking and disturbance rejection) generated from the high-level control requirements.

The high-level design was carried out using an LQ with frequency-shaping technique [6]. Figure 2 displays the stability and tracking performance specifications resulting from the highlevel design. The tracking performance robustness measure corresponds to a 5% degradation from nominal.

The row interaction measures for the airflow and gas generator subsystems are shown in Figures 3 and 4. The propulsion subsystems designs were carried out using an LQ output feedback method. Figures 5 and 6 show a comparison of the bound on compensator gains and the actual subsystem compensator transfer function norms. As seen, both constraints are violated. However, the closed-loop system has been shown to be stable through simulation. Note that Theorem 2 was a sufficient condition for stability. The condition is clearly conservative. These initial results are extremely encouraging and motivate effort to develop less conservative measures.

IV. CONCLUSIONS

This paper has described a method for robust control of decentralized multivariable systems. The method relies upon generation of a set of control design specifications for the subsystems. Two types of specifications have been developed. They deal with stability and performance robustness properties of the closed-loop system. The first type of specification is an accuracy/bandwidth requirement and the second is a compensator bound requirement. subsystem controls may be designed using any multivariable control technique. The utility of the results has been demonstrated by application to an integrated flight propulsion control example. The results, even though conservative, are extremely useful. Less conservative design specifications would be very desirable. Also, design specifications which allow for communication among controllers need to be developed. Research in these directions is continuing.

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REFERENCES

- [1] Safonov, M.G., "Tight Bounds on the Response of Multivariable Systems with Component Uncertainty," <u>Proc. Allerton Conf.</u>, pp.451-460, 1978.
- [2] Bennett, W.H. and J.S. Baras, "Block Diagonal Dominance and Design of Decentralized Compensators," Report SRR 83-2, Dept. of Electrical Engineering, University of Maryland, 1983.
- [3] Limebeer, D.J.N. and Y.S. Hung, "Robust Stability of Interconnected Systems," <u>IEEE Trans. Auto. Control</u>, Vol. AC-28, No. 6, pp.710-716, June 1983.
- [4] Ohta, Y., D.D. Siljak, and T. Matsumoto, "Decentralized Control Using Quasi-Block Diagonal Dominance of Transfer Function Matrices," <u>Proc.</u> 22nd Allerton Conf., October 1984.
- 22nd Allerton Conf., October 1984.
 [5] Joshi, D.S., Shaw, P.D., Hodgkinson, J., Rock, S.M., Vincent, J.H., and Fisk, W.S., "A Design Approach to Integrated Flight and Propulsion Control," SAE 831482, Long Beach, California, 1983.
- [6] Vincent, J.H., "Direct Incorporation of Flying Qualities Criteria into Multivariable Flight Control Design," AIAA-84-1830-CP, AIAA Guidance and Control Conference, Seattle, Washington, August 1984.

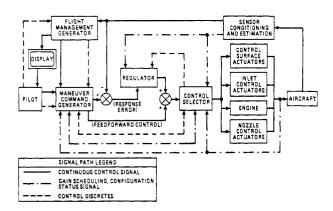


Figure 1 Integrated Flight Propulsion Control System Structure

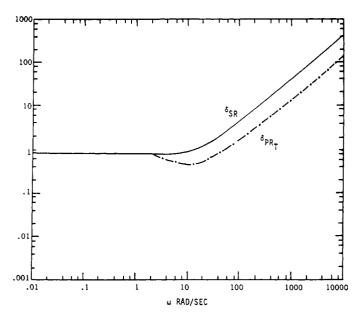


Figure 2 Stability and Performance Robustness Specification

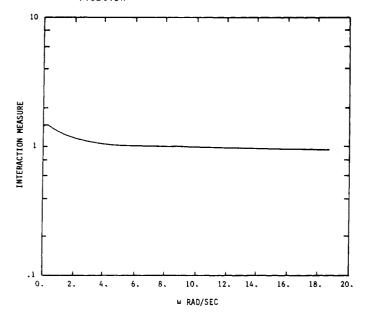
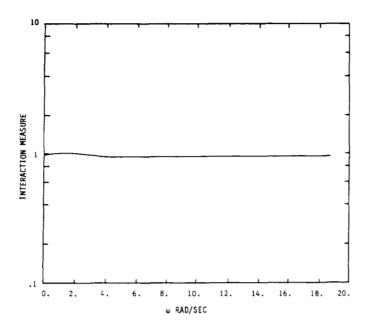


Figure 3 Row Interaction Measure for Airflow Subsystem



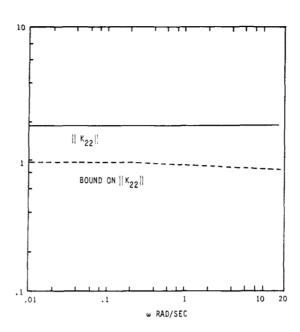
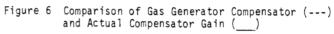


Figure 4 Row Interaction Measure for Gas Generator Subsystem



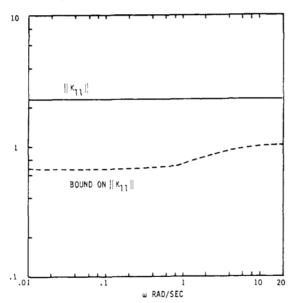


Figure 5 Comparison of Airflow Compensator Bound (---) and Actual Compensator Gain (___)